

# Unified criteria for multipartite quantum nonlocality

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Wiseman and co-workers (Phys. Rev. Lett. **98**, 140402, 2007) proposed a distinction between the nonlocality classes of Bell nonlocality, steering and entanglement based on whether or not an overseer *trusts* each party in a bipartite scenario where they are asked to demonstrate entanglement. Here we extend that concept to the multipartite case and derive inequalities that progressively test for those classes of nonlocality, with different thresholds for each level. This framework includes the three classes of nonlocality above in special cases and introduces a family of others.

The Einstein-Podolsky-Rosen (EPR) paradox [1] revealed an incompatibility between local causality and the completeness of quantum mechanics. Bell later discovered that quantum mechanics (QM) can violate certain inequalities that are predicted by all Local Hidden Variable (LHV) theories [2]. Schrödinger [3] introduced the term *steering* to describe the “spooky action at a distance” envisaged by EPR where an agent can apparently “steer” a distant quantum state, and the term *entanglement* for the nonfactorisable property possessed by all states that demonstrate this phenomenon.

Entanglement, EPR paradox, and Bell’s nonlocality were seen as requiring the same resources until Werner [4] discovered that not all entangled states can violate a Bell inequality. At a similar time, experiments by Ou *et al.* [5] demonstrated the EPR paradox, for measurements which would admit a LHV model but based on criteria specific to the EPR paradox [6], thus further suggesting that the three types of nonlocality embody different physics.

This idea wasn’t fully formalised until recently, when Wiseman, Jones, and Doherty (WJD) [7] presented a definition of steering as violation of what they termed a *Local Hidden State* (LHS) model. They showed that the states exhibiting steering are a strict *subset* of the entangled states, and a strict *superset* of the Bell-nonlocal states (those violating a LHV model). In Ref. [8], the EPR paradox and steering were shown to be equivalent concepts, and EPR-steering criteria were defined as any experimental consequences, usually in the form of inequalities, of the LHS model. Based on that work, Saunders *et al.* [9] experimentally demonstrated EPR steering for a Bell-local state, thus confirming EPR steering as a distinct class of nonlocality.

The LHS approach provides a rigorous framework from which to derive a multitude of criteria for the “intermediate” EPR-steering nonlocality—in scenarios *different* to the one originally considered by EPR and Schrödinger. This allows fresh investigations of a form of nonlocality which has a profound historical significance but about which relatively little is known [10].

However, an important scenario remains unexplored. This prior work was limited to the bipartite case,

as in EPR’s original argument. WJD presented an information-theoretic interpretation for the distinction between entanglement, EPR steering and Bell nonlocality, based on whether an overseer *trusts* each of two parties in a task where they are asked to demonstrate entanglement. That approach motivates our extension to multiple parties.

Further motivation exists, as the  $N$ -party scenario displays very interesting behaviour. Greenberger, Horne and Zeilinger (GHZ) [11] showed that multipartite “GHZ states” exhibit extreme forms of nonlocality, which Mermin, followed by Ardehali, Belinski and Klyshko (MABK) [12], characterised by inequalities whose violation by QM increases exponentially as  $2^{(N-1)/2}$  with  $N$ . Werner and Wolf [13] later proved the MABK inequalities to be those violated by “the widest margin in quantum theory”, for the two-setting, two-outcome experiments. Surprisingly, this multi-party Bell nonlocality is stable against loss [14]. Cabello *et al.* [15] showed that above a critical detection efficiency  $\eta_{crit} = N/(2N-2)$  there is no LHV model to describe violations of Mermin’s inequality.

In this paper, we introduce a family of locally causal models involving  $N$  parties,  $T$  of which are trusted, and derive inequalities to test for their failure. Entanglement, EPR steering and Bell nonlocality apply to special cases where all, one or no parties are trusted. These inequalities take the form of MABK but with different thresholds for each level of nonlocality.

We prove that violation of the EPR steering inequalities grows exponentially, as  $2^{(N-2)/2}$ , for continuous variable (CV) measurements, and as  $2^{(N-1)/2}$ , for dichotomic measurements. Unlike the Bell violation, however, the dichotomic maximal violation manifests, for all  $N$ , for *both* of two famous choices of measurement settings: EPR and GHZ’s that gives a perfect correlation between outcomes, and Bell’s, that gives statistical correlation only. Furthermore, we show as a consequence that EPR steering is more resistant to loss than Bell nonlocality.

We begin by introducing some notation: we consider  $N$  spatially separated parties labelled by  $j$  who can choose between a number of experiments. We label the experiments by lower-case letters  $x_j \in \mathcal{M}_j$ . The respective

outcomes are labelled by upper-case letters  $X_j \in \mathcal{O}_{x_j}$ . A sufficient specification of the features of the preparation procedure which are explicitly *known* by the experimenters is labelled by  $\kappa$ ; a sufficient specification of *any* (possibly unknowable and thus “hidden”) variables which may be relevant to the experiments considered is labelled by  $\lambda$ . Whenever an equation involving those variables appears, it is implicitly assumed that the equation holds for all values of the variables.

Before continuing, we summarise the nonlocality hierarchy for bipartite systems [7]. The strongest form of nonlocality is *Bell nonlocality*, in which LHV models are falsified [2]. LHV models require that probabilities for joint measurements at sites  $A$  (Alice) and  $B$  (Bob) can be written in the factorisable form:

$$P(X_A, X_B | x_A, x_B, \kappa) = \int d\lambda P(\lambda | \kappa) P(X_A | x_A, \lambda, \kappa) P(X_B | x_B, \lambda, \kappa). \quad (1)$$

A *further* assumption is to require that Bob’s “local state” be *quantum*: ie. there must be a quantum state  $\rho_{\lambda, \kappa}$  such that for all outcomes  $X_B$  of all measurements  $x_B$

$$P(X_B | x_B, \lambda, \kappa) = \text{Tr}[E_{X_B} \rho_{\lambda, \kappa}] \equiv P_Q(X_B | x_B, \lambda, \kappa), \quad (2)$$

where  $E_{X_B}$  is the POVM element associated with  $X_B$ . With this assumption, we arrive at the asymmetric LHS model of WJD [7]. *EPR Steering* (of Bob’s state by Alice) arises when this model fails [7, 8]. *Entanglement* arises as a failure of the Quantum Separable (QS) model,  $P(X_A, X_B) = \int d\lambda P(\lambda) P_Q(X_A | \lambda) P_Q(X_B | \lambda)$ , in which one assumes quantum states for *both* systems.

Now consider the following task [7]: Charlie wants to demonstrate entanglement between  $N$  parties. Initially, he will be satisfied if their correlations cannot be written as a quantum separable (QS) model (leaving  $\kappa, x_A, x_B$  henceforth implicit)

$$P(X_1, \dots, X_N) = \int d\lambda P(\lambda) \prod_{j=1}^N P_Q(X_j | \lambda). \quad (3)$$

Suppose now that Charlie trusts the first  $T$  agents (and their apparatus), but not the remaining  $N-T$  (or their apparatus). That is, he does not trust that the measurement outcomes reported by the untrusted group correspond to the quantum observables they report to have measured. In this case, the QS model may be violated even without any entanglement. However, if the observed correlations cannot be reproduced by a model of form

$$P(X_1, \dots, X_N) = \int d\lambda P(\lambda) \prod_{j=1}^T P_Q(X_j | \lambda) \prod_{j=T+1}^N P(X_j | \lambda), \quad (4)$$

with the outcomes of the untrusted parties given by arbitrary (not necessarily quantum) LHV distributions

$P(X_j | \lambda)$ , Charlie will be convinced they share entanglement, since then no locally causal model exists that could be used by an untrusted party to generate the statistics. We denote the multipartite LHS model (4) with  $T$  trusted sites and  $N-T$  untrusted sites by  $\text{LHS}(T, N)$ . Violation of the  $\text{LHS}(T, N)$  model confirms entanglement in the presence of  $N-T$  untrusted sites.

Violation of a  $\text{LHS}(N, N)$  model is equivalent to a standard entanglement test, while violation of  $\text{LHS}(0, N)$  model implies Bell nonlocality. Following Ref. [7], violation of a  $\text{LHS}(1, 2)$  model is a demonstration of EPR steering. For  $T > 1$ , failure of  $\text{LHS}(T, N)$  implies entanglement, but not necessarily EPR steering. The most interesting case is violation of  $\text{LHS}(1, N)$ . This violation can only occur if EPR steering exists for some bipartition between the trusted site and the untrusted sites taken as a group, or to the violation of a LHV model among the non-trusted sites (which in turn implies EPR steering). However, in these cases we cannot interpret the situation as the state in a specific site being steered by the others, so we refer to violation of  $\text{LHS}(1, N)$  as a multipartite demonstration of EPR steering.

We now turn to derive inequalities to demonstrate failure of each member of the family of locally causal models (4). Following [12, 16], we construct complex functions  $F_j^\pm = X_j \pm iY_j$  of measurement outcomes  $X_j, Y_j$  at each site  $j$ . For any  $\text{LHS}(T, N)$  model (4),  $\langle \prod_{j=1}^N F_j^{s_j} \rangle = \int d\lambda P(\lambda) \prod_{j=1}^N \langle F_j^{s_j} \rangle_\lambda$ , where  $s_j \in \{-, +\}$ . Here  $\langle F_j^\pm \rangle_\lambda = \langle X_j \rangle_\lambda \pm i \langle Y_j \rangle_\lambda$  and  $\langle X_j \rangle_\lambda = \sum_{X_j} P(X_j | \lambda) X_j$ , with  $P(X_j | \lambda) = P_Q(X_j | \lambda)$  for the trusted parties,  $1 \leq j \leq T$ . From the variance inequality it then follows that

$$|\langle \prod_{j=1}^N F_j^{s_j} \rangle|^2 \leq \int d\lambda P(\lambda) \prod_{j=1}^N |\langle F_j^{s_j} \rangle_\lambda|^2. \quad (5)$$

Since  $|\langle F_j^\pm \rangle_\lambda|^2 = \langle X_j \rangle_\lambda^2 + \langle Y_j \rangle_\lambda^2$ , it follows from the non-negativity of variances that for any LHV (untrusted) state:  $|\langle F_j^\pm \rangle_\lambda|^2 \leq \langle X_j^2 \rangle_\lambda + \langle Y_j^2 \rangle_\lambda$ . For a local *quantum* (trusted) state, quantum uncertainty relations impose further restrictions. We consider uncertainty relations of the form  $\Delta^2 X_j + \Delta^2 Y_j \geq C_j$ , where  $C_j$  depends on the operators associated to  $x_j$  and  $y_j$ . Substituting on (5) we obtain a family of nonlocality criteria:

$$|\langle \prod_{j=1}^N F_j^{s_j} \rangle| \leq \left\langle \prod_{j=1}^T (X_j^2 + Y_j^2 - C_j) \prod_{j=T+1}^N (X_j^2 + Y_j^2) \right\rangle^{1/2}. \quad (6)$$

So far no assumption was made about the measurements  $x_j, y_j$ . We now consider two cases: continuous and dichotomic outcomes. For the continuous case, we assume position-momentum conjugation relations  $[\hat{x}_j, \hat{y}_j] = i$  for the trusted sites, which imply the local uncertainty relation  $\Delta^2 X_j + \Delta^2 Y_j \geq 1$ , i.e.,  $C_j = 1$ .

Thus the LHS model (4) implies

$$|\langle \prod_{j=1}^N F_j^{s_j} \rangle| \leq \left\langle \prod_{j=1}^T (X_j^2 + Y_j^2 - 1) \prod_{j=T+1}^N (X_j^2 + Y_j^2) \right\rangle^{1/2}. \quad (7)$$

With  $T = N$ , we obtain the entanglement criterion of Hillery and Zubairy [17]; with  $T = 0$ , we obtain the Bell inequality of Cavalcanti *et al.* [16]. We now show these inequalities can be violated by QM. Using quadrature operators  $\hat{x}_j = (\hat{a}_j + \hat{a}_j^\dagger)/\sqrt{2}$ ,  $\hat{y}_j = i(\hat{a}_j^\dagger - \hat{a}_j)/\sqrt{2}$ , where  $\hat{a}_j^\dagger, \hat{a}_j$  are bosonic creation/annihilation operators ( $[\hat{a}_j, \hat{a}_k^\dagger] = \delta_{j,k}$ ), we obtain  $F_j^+ = \sqrt{2}\hat{a}_j^\dagger$  and  $F_j^- = \sqrt{2}\hat{a}_j$ , and  $(\hat{x}_j^2 + \hat{y}_j^2) = 2\hat{a}_j^\dagger \hat{a}_j + 1 = 2\hat{n}_j + 1$ ,  $\hat{n}_j$  being the number operator for each site. Symbolising  $\hat{a}^+ = \hat{a}^\dagger$  and  $\hat{a}^- = \hat{a}$ , the inequalities (7) will be violated when

$$|\langle \hat{a}_1^{s_1} \dots \hat{a}_N^{s_N} \rangle| > \langle \prod_{j=1}^T \hat{n}_j \prod_{j=T+1}^N (\hat{n}_j + 1/2) \rangle^{1/2}. \quad (8)$$

Consider the  $N$  sites prepared in a GHZ-type state [11]

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes r} |1\rangle^{\otimes N-r} + e^{i\phi} |1\rangle^{\otimes r} |0\rangle^{\otimes N-r}), \quad (9)$$

where  $r \in \{1, \dots, N\}$ ;  $|n\rangle^{\otimes r} \equiv \bigotimes_{j=1}^r |n\rangle_j$ ;  $|n\rangle^{\otimes N-r} \equiv \bigotimes_{j=r+1}^N |n\rangle_j$  and  $|n\rangle_j$  are the eigenstates of  $\hat{n}_j$ . Taking  $\phi = 0$ , the left-side of (8) is nonzero when  $s_j = +$  for all  $j \leq r$  and  $s_j = -$  for all  $j > r$ ; or vice-versa. For those parameters,  $|\langle \prod_{k=1}^N \hat{a}_k^{s_k} \rangle| = 1/2$ . For the right-side, note that the ordering of the trusted sites does not need to coincide with the ordering of the sites on Eq. (9). Inspecting Eq. (8) we see that the trusted sites will annihilate the terms of (9) when their corresponding state is  $|0\rangle$ . Thus for all  $T \geq 2$ ,  $r \neq 0$ , we can choose the ordering on state (9) such that both terms will give zero contribution and inequality (7) will be violated by the same amount in all cases. For  $T = 1$ ,  $r \neq 0$ , the right-side is  $(3^{r-1}/2^N)^{1/2}$  and we can violate the inequality for  $N \geq 3$ . For the optimal case of  $r = 1$ , the ratio of left to right side becomes  $2^{N-2/2}$ , an exponential increase of EPR-steering with  $N$ . Bell nonlocality ( $T = 0$ ) requires  $N > 9$ , with  $r = N/2$  optimal, as shown in [16, 18].

We now examine the dichotomic case using qubits. We choose  $\hat{x}_j = \sigma_j^\theta$ ,  $\hat{y}_j = \sigma_j^{\theta+\pi/2}$  where  $\sigma_j^\theta = \sigma_j^x \cos \theta + \sigma_j^y \sin \theta$ , and  $\sigma_j^{x/y}$  are the Pauli spin operators ( $\theta$  can be different for each site). Using the local uncertainty relation  $\Delta^2 \sigma_j^x + \Delta^2 \sigma_j^y \geq 1$  [19] ( $C_j = 1$ ) for the trusted sites and the identity  $(\sigma_j^\theta)^2 = 1$ , inequality (6) becomes

$$|\langle \prod_{j=1}^N F_j^{s_j} \rangle| \leq 2^{(N-T)/2}. \quad (10)$$

Defining the Hermitian parts of the operator product by  $\prod_{j=1}^N F_j^{s_j} = \text{Re}\Pi_N + i\text{Im}\Pi_N$ , inequality (10) implies

$$\langle \text{Re}\Pi_N \rangle, \langle \text{Im}\Pi_N \rangle \leq 2^{(N-T)/2}, \quad (11)$$

$$\langle \text{Re}\Pi_N \rangle + \langle \text{Im}\Pi_N \rangle \leq 2^{(N-T+1)/2}, \quad (12)$$

For  $T = N$  these reduce to the separability inequalities of Roy [20]. These inequalities take the form of the MABK inequalities, but for the Bell case ( $T = 0$ ), a stronger bound can be found [12]: for *odd*  $N$  only,

$$\langle \text{Re}\Pi_N \rangle, \langle \text{Im}\Pi_N \rangle \leq 2^{(N-1)/2}, \quad (13)$$

(which is Mermin's inequality), and for *even*  $N$  only,

$$\langle \text{Re}\Pi_N \rangle + \langle \text{Im}\Pi_N \rangle \leq 2^{N/2}. \quad (14)$$

(which is the Ardehali-CHSH inequality [2, 12]). The reason for the different bounds is that in the Bell case, the set of values of  $\langle \prod_j F_j^{s_j} \rangle$  that is defined by all convex combinations of the classical extreme points (i.e., all LHV models) is a square in the complex plane [13], and thus the edges of the square provide tighter bounds than that given by the maximum modulus of  $\langle \prod_j F_j^{s_j} \rangle$ . When one or more parties are treated as having local quantum states, however, this set becomes a circle, due to the continuum of pure states allowed by quantum mechanics, and thus the bound given by (10) is tight.

Defining  $|0\rangle$  and  $|1\rangle$  now as eigenstates of  $\sigma^z$ , the GHZ state (9) ( $r = N$ ) violates (11-14) by the maximum amount for QM [13]. This occurs for Mermin-type inequalities (11), (13) for  $F_j = \sigma_j^x \pm i\sigma_j^y$  ( $j = 1, \dots, N$ ), which is the case of the EPR-Bohm and GHZ paradoxes [1, 11] that yield perfect correlations between spatially separated spins. The quantum prediction is:

$$\langle \text{Re}\Pi_N \rangle, \langle \text{Im}\Pi_N \rangle = 2^{N-1}. \quad (15)$$

With  $F_j = \sigma_j^x - i\sigma_j^y$  ( $j \neq N$ ),  $F_N = \sigma^{\pi/4} + i\sigma^{\pi/4}$  we get the maximum quantum prediction for the Ardehali-type inequalities (12), (14):

$$\langle \text{Re}\Pi_N \rangle + \langle \text{Im}\Pi_N \rangle = 2^{N-1/2}. \quad (16)$$

The ratio of left to right side of (11) and (12) is thus  $S_N = 2^{(N+T-2)/2}$ , an exponential growth for all  $T$ .

EPR steering is shown when the inequalities (11) and (12) with  $T = 1$  are violated. Interestingly, while the ratio  $S_N = 2^{(N-1)/2}$  is unchanged from the optimal MABK case, there is the new feature that this ratio is achieved for *both* statistical and perfect correlations, i.e. via *both* inequalities (11) and (12). Thus, additional EPR-steering criteria different to the MABK inequalities follow from (12) when  $N$  is odd, and from (11) when  $N$  is even: e.g. EPR steering is confirmed if  $|\langle \sigma_1^x \sigma_2^x \rangle - \langle \sigma_1^y \sigma_2^y \rangle| > \sqrt{2}$ . As shown by Roy [20], entanglement criteria also follow from both inequalities when  $T = N$ : e.g. entanglement is confirmed for the CHSH Bell inequality [2] with a lower threshold:  $|\langle \sigma_1^x \sigma_2^{x'} \rangle - \langle \sigma_1^y \sigma_2^{y'} \rangle + \langle \sigma_1^y \sigma_2^{x'} \rangle + \langle \sigma_1^x \sigma_2^{y'} \rangle| > \sqrt{2}$ .

Crucial in many nonlocality experiments is the effect of loss [15]. Following [21], we model loss with a beam-splitter and calculate moments of detected fields, using  $a_{\text{det}} = \sqrt{\eta}a + \sqrt{1-\eta}a_{\text{vac}}$ . Here  $a_{\text{vac}}$  is the operator for a vacuum reservoir mode into which quanta are lost.

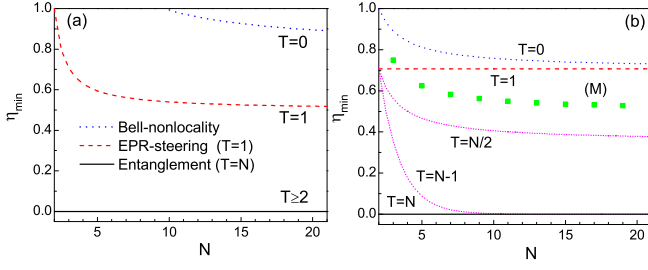


Figure 1. Efficiency  $\eta$  sufficient to demonstrate nonlocality using (7) and (11)–(14) with states (9). (a) CV case. (b) Dichotomic case. For  $T \geq 1$ ,  $\eta_{\min}$  values are for *both* inequalities (11) and (12), but for  $T = 0$  apply *only* to (13) for odd  $N$ , and to (14) for even  $N$ . Points (M) give  $\eta_{\min}$  necessary and sufficient for failure of LHV using Mermin’s inequality (13).

To summarise the calculation, we start with the continuous variable case:  $[a_{det}, a_{det}^\dagger] = 1$  and inequalities (7-8) still apply. The left-side of (8) becomes  $\eta_t^{T/2} \eta_u^{(N-T)/2} / 2$  where  $\eta_t$  ( $\eta_u$ ) are efficiencies at trusted (untrusted) sites respectively. Examining the right-side and optimising  $r$  of (9), we can detect violation of  $\text{LHS}(T, N)$  for  $T \geq 2$ , with  $r \geq 2$  and *any* nonzero efficiencies  $\eta_t, \eta_u$ . To test EPR steering ( $T = 1$ ) we need (with  $r \geq 1$ )  $\eta_u^{N-1} > 2^{r-N+1}(\eta_u + 1/2)^{r-1}$ , which reduces to

$$\eta_u > 2^{1/(N-1)} / 2 \quad (17)$$

for  $r = 1$ . We note an interesting asymmetry: there is sensitivity to loss at the *untrusted* sites only, an effect noticed for the bipartite EPR paradox [23]. The limiting efficiency of 50% required to demonstrate EPR steering is much more accessible than that required for Bell nonlocality [16] ( $\eta_u > (1 + \sqrt{5})/4 \approx 81\%$  as  $N \rightarrow \infty$ ).

For the qubit case, violation of inequalities (11)–(14) is the same for EPR steering and Bell nonlocality. However, analysis reveals that in important scenarios the former will be less sensitive to loss. As GHZ states have been prepared for qubits in optical polarisation states [22], we model loss with beam-splitters as in the continuous-variable case. Denoting mode operators by  $a_\pm$ , we introduce Schwinger spins [24] for each site  $j$  (subscripts  $j$  for operators are implicit):  $s^z = a_+^\dagger a_+ - a_-^\dagger a_-$ ,  $s^x = (a_+^\dagger a_- + a_+ a_-^\dagger)$ ,  $s^y = (a_+^\dagger a_- - a_+ a_-^\dagger)/i$ , and  $s^2 = n(n+2)$ , where  $n = a_+^\dagger a_+ + a_-^\dagger a_-$  is the number operator for each site. We map the components of the GHZ state (9) into the  $\pm 1$  eigenstates of  $s^z$ :  $|0\rangle_j \rightarrow |0\rangle_{+j}|1\rangle_{-j}$  and  $|1\rangle_j \rightarrow |1\rangle_{+j}|0\rangle_{-j}$ . With loss of a photon at site  $j$ , the local state becomes  $|0\rangle_{+j}|0\rangle_{-j}$ , and  $s^z = 0$ . The Bell inequalities (13)–(14) still follow from (6) with the extra ‘0’ outcome, since  $X^2, Y^2 \leq 1$ . For  $T > 0$ , we use the uncertainty relation  $\Delta^2 s^x + \Delta^2 s^y \geq \Delta^2 n - \Delta^2 s^z + 2\langle n \rangle$  [24] to obtain  $|\langle F_j^\pm \rangle_\lambda|^2 \leq \eta_t^2$  [25]. Inequality (6) becomes

$$|\langle \prod_{j=1}^N F_j^{s_j} \rangle| \leq 2^{(N-T)/2} \eta_t^T. \quad (18)$$

With state (9) ( $r = N$ ), we need  $\eta_u > 2^{(2-N-T)/(2(N-T))}$  to detect failure of the  $\text{LHS}(T, N)$  model via these inequalities. EPR steering ( $T = 1$ ) requires  $\eta_u > 1/\sqrt{2}$  (Fig.1 (b)). We see that for  $N = 2, 3$ , EPR steering is detectable with lower efficiency than the  $\eta_{crit} > N/(2N-2)$  necessary and sufficient for incompatibility of the measurements of (11) with a LHV model [15].

In conclusion, we have presented a unified framework to derive criteria for a family of nonlocality models based on differing levels of trust on different parties. Those criteria are sufficient to demonstrate Bell nonlocality, EPR steering and entanglement as special cases. The criteria follow from one general proof, with different bounds for each type of nonlocality, thus clarifying the relationship between them. We note that violation of the inequalities presented here (as for MABK inequalities) are sufficient but not necessary conditions for a given quantum state to display the corresponding type of nonlocality. This is evident on examining the criteria for the mixed Werner states [4], for which the boundary for entanglement and steering is known [7, 26]. A promising avenue is to search for criteria involving more than two settings.

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- [1] A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. **47**, 777 (1935).
  - [2] J.S. Bell, Physics **1**, 195 (1964); Epistemological Letters **9**, 11 (1976). J.F. Clauser, M.A. Horne, A. Shimony, and R.A. Holt, Phys. Rev. Lett. **23**, 880 (1969).
  - [3] E. Schrodinger, Proc. Camb. Phil. Soc. **31**, 553 (1935); Naturwissenschaften, **23**, 807 (1935).
  - [4] R.F. Werner, Phys. Rev. A **40**, 4277 (1989).
  - [5] Z.Y. Ou et al, Phys. Rev. Lett. **68**, 3663 (1992).
  - [6] M.D. Reid, Phys. Rev. A **40**, 913 (1989).
  - [7] H.M. Wiseman, S.J. Jones and A.C. Doherty, Phys. Rev. Lett. **98**, 140402 (2007); S. J. Jones H. M. Wiseman, and A. C. Doherty, Phys. Rev. A **76**, 052116 (2007).
  - [8] E.G. Cavalcanti et al., Phys. Rev. A **80**, 032112 (2009).
  - [9] D.J. Saunders et al., Nature Physics **6**, 845 (2010).
  - [10] J. Oppenheim and S. Wehner, Science **19**, 1072 (2010).
  - [11] D.M. Greenberger, M. Horne and A. Zeilinger, in *Bell’s Theorem, Quantum Theory and Conceptions of the Universe*, M. Kafatos, ed., (Kluwer, Dordrecht, The Netherlands), (1989). N.D. Mermin, Phys. Today **43**, 9 (1990).
  - [12] N.D. Mermin, Phys. Rev. Lett. **65**, 1838 (1990). M. Ardehali, Phys. Rev. A **46**, 5375 (1992). A.V. Belinskii and D.N. Klyshko, Physics-Uspekhi **36**, 654 (1993).
  - [13] R.F. Werner and M.M. Wolf, Phys. Rev. A **64**, 032112 (2001).
  - [14] S.L. Braunstein and A. Mann, Phys. Rev. A **47**, R2427 (1993).
  - [15] A. Cabello et al, Phys. Rev. Lett. **101**, 120402 (2008).
  - [16] E.G. Cavalcanti et al, Phys. Rev. Lett. **99**, 210405 (2007).
  - [17] M. Hillery and M.S. Zubairy, Phys. Rev. Lett. **96**, 050503 (2006).
  - [18] Q.Y. He et al., Phys. Rev. A **81**, 062106 (2010).
  - [19] H.F. Hofmann and S. Takeuchi, Phys. Rev. A **68**, 032103 (2003).

- [20] S.M. Roy, Phys. Rev. Lett. **94**, 010402 (2005).
- [21] B. Yurke and D. Stoler, Phys. Rev. Lett. **57**, 13 (1986).
- [22] C-Y Lu et al., Nature Physics **3**, 91 (2007).
- [23] M.D. Reid et al., Rev. Mod. Phys. **81**, 1727 (2009).
- [24] G. Toth, Phys. Rev. A **69**, 052327 (2004).
- [25] We rearrange  $\Delta^2 s_j^x + \Delta^2 s_j^y \geq \Delta^2 n_j - \Delta^2 s_j^z + 2\langle n_j \rangle$  and substitute  $s^2 = n(n+2)$  to get  $\langle s^x \rangle^2 + \langle s^y \rangle^2 \leq \langle n \rangle^2 - \langle s^z \rangle^2 \leq \langle n \rangle^2 = \eta_t^2$ .
- [26] A. Peres, Phys. Rev. Lett. **76**, 1413 (1996).